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Error Theory of Intersection Photogrammetry

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ABERDEEN PROVING GROUND, MARYLAND

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ABERDEEN PROVING GROUND, MARYLAND

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ERROR THEORY OF INTERSECTION PHOTOGRAMMETRY

ABSTRACT

Formulas are developed for evaluating the propagation of the various errors present in a system of photogrammetric cameras in triangulating the space position of a photographed point.

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I. INTRODUCTION

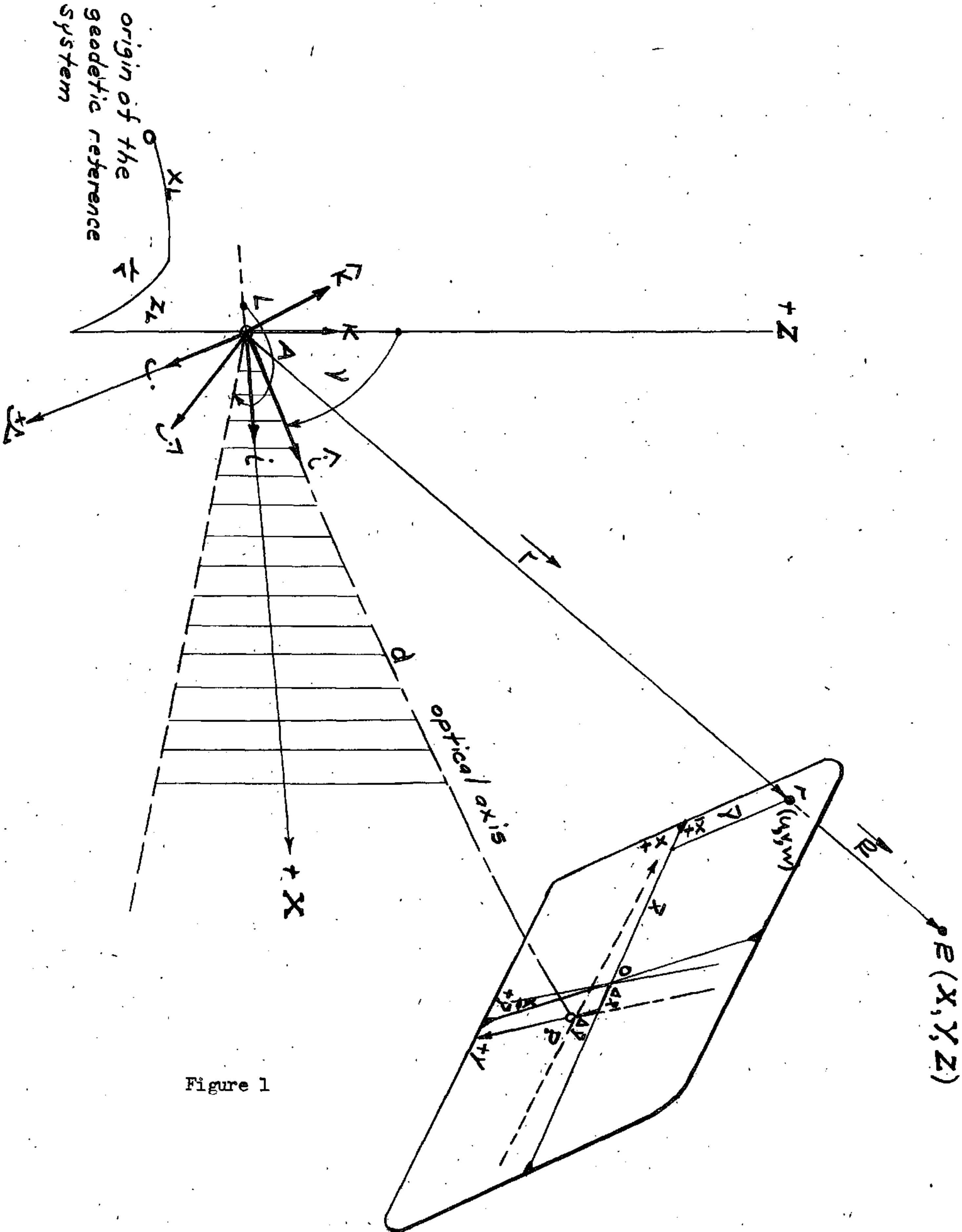
If at different points on the surface of the earth photogrammetric cameras are established in order to record points, and if the orientation of the photographs is known and the corresponding plate coordinates are measured, then the spatial position of such points may be computed. Besides the errors in the geodetic reference data of the stations, the errors of the parameters of orientation and of the measured plate coordinates contribute to errors in the computation of the individual spatial direction to any target point, and consequently lead to errors of the spatial coordinates of the target. This paper deals with the determination of the influence of the individual residuals and gives formulas for the combined effect of these errors.

A study of the error theory of intersection photogrammetry must follow steps which are typical of the application of the method. A basic characteristic of the triangulation method is the fact that each camera in the course of the reductions is treated as an independent unit up to the moment when the direction to a target is obtained. Such a direction may be expressed by two position angles (e.g., Azimuth and Zenith distance). Only in the final step in the triangulation procedure will the directions from the individual cameras be combined for the determination of the most probable point of intersection. Consequently, the problem is to develop first the error theory of an individual photogrammetric camera, and later to determine the propagation of the individual station errors in the triangulation procedure.

II. THE ERROR THEORY OF A PHOTOGRAMMETRIC CAMERA

1. The orientation of a photogrammetric camera

We introduce a rectangular cartesian coordinate system in such a way that its origin is the center of projection and its orientation is consistent with the local geodetic reference system. Thus, the X, Y, plane represents the horizontal plane at the station. The X-axis may be chosen positive to the north. The Z-axis is perpendicular to the X, Y axes, positive to the zenith. Further, we always consider the interior and exterior orientation of a plate simultaneously; that is, we consider the plate in its spatial position. Consequently, the plate coordinate system - primarily a plane system - will be introduced as another rectangular cartesian spatial system with its origin again at the center of projection. The plate is introduced as a diapositive. Figure 1 shows the situation.



origin of the geodetic reference system
+Z
+X
optical axis
P(X,Y,Z)

Figure 1

L is the center of projection. Its geodetic coordinates denoted by x_L, y_L and z_L may be expressed in any suitable rectangular coordinate system which need not necessarily be a cartesian system. In L we establish two rectangular cartesian systems. The X-Y-Z system, denoted as the station system, is expressed by the unit vectors i, j, k. In this system, a spatial target point R is determined by the coordinates X, Y, Z. Its image point is r, denoted by the coordinates u, v, w respectively. The system is oriented according to the geodetic reference system. The other cartesian system expressed by the unit vectors $\hat{i}$, $\hat{j}$, $\hat{k}$ denotes the camera system. Its orientation relative to the station system is expressed by the three elements of exterior orientation denoted by the azimuth angle A (counted clockwise from the south), the tilt angle ν of the plate perpendicular, and by the swing angle κ of the fiducial mark system. The length of the plate perpendicular - the principal distance - is denoted by d. Δx and Δy are the coordinates of the principal point P in any rectangular plane plate coordinate system established by fiducial marks and denoted by $\bar{x}$ and $\bar{y}$. x and y are the plane plate coordinates of the image point r in the oriented x, y system.

Further, we introduce the image vector $\vec{r}$ and the object vector $\vec{R}$ respectively.

Then:

$$\vec{r} = iu + jv + kw = +\hat{i}d - \hat{j}x - \hat{k}y \tag{1}$$

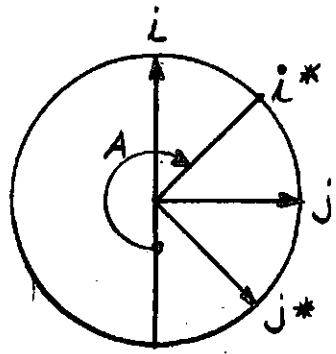
$$\text{and } \vec{R} = iX + jY + kZ$$

The transformation of the triple vector $\hat{i}$, $\hat{j}$, $\hat{k}$ into the triple vector i, j, k is obtained from the transformation matrix

$$\begin{array}{c|ccc} & \hat{i} & \hat{j} & \hat{k} \\ \hline i & i\hat{i} & i\hat{j} & i\hat{k} \\ j & j\hat{i} & j\hat{j} & j\hat{k} \\ k & k\hat{i} & k\hat{j} & k\hat{k} \end{array}$$

The individual transformation matrices are:

1.) for a rotation in azimuth denoted temporarily by*:



rotation clockwise positive

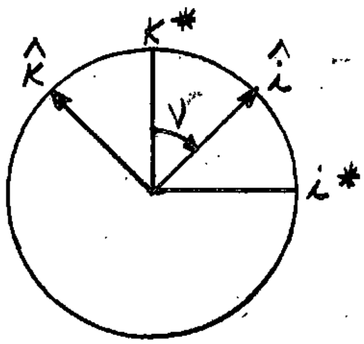
$i = -i^* \cos A + j^* \sin A$

$j = -i^* \sin A - j^* \cos A$

$k = k^*$

$$\begin{array}{c|c} & \begin{matrix} i^* & \quad j^* & \quad k^* \end{matrix} \\ \hline \begin{matrix} i \\ j \\ k \end{matrix} & \begin{pmatrix} -\cos A & +\sin A & 0 \\ -\sin A & -\cos A & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

2.) for a rotation in zenith distance, positive downwards:



$i^* = \hat{i} \sin \nu - \hat{k} \cos \nu$

$j^* = \hat{j}$

$k^* = \hat{i} \cos \nu + \hat{k} \sin \nu$

$$\begin{array}{c|c} & \begin{matrix} \hat{i} & \quad \hat{j} & \quad \hat{k} \end{matrix} \\ \hline \begin{matrix} i^* \\ j^* \\ k^* \end{matrix} & \begin{pmatrix} \sin \nu & 0 & -\cos \nu \\ 0 & 1 & 0 \\ \cos \nu & 0 & \sin \nu \end{pmatrix} \end{array}$$

and the combined transformation matrix is:

$$\begin{array}{c|c} & \begin{matrix} \hat{i} & \quad \hat{j} & \quad \hat{k} \end{matrix} \\ \hline \begin{matrix} i \\ j \\ k \end{matrix} & \begin{pmatrix} -\cos A \sin \nu & +\sin A & +\cos A \cos \nu \\ -\sin A \sin \nu & -\cos A & +\sin A \cos \nu \\ +\cos \nu & 0 & +\sin \nu \end{pmatrix} \end{array}$$

Or:

$$\hat{i} = -i \cos A \sin \nu - j \sin A \sin \nu + k \cos \nu$$

$$\hat{j} = +i \sin A - j \cos A \qquad (2)$$

$$\hat{k} = +i \cos A \cos \nu + j \sin A \cos \nu + k \sin \nu$$

with formulas (1) and (2), we obtain:

$$+\hat{i}d = -id \cos A \sin \nu - jd \sin A \sin \nu + kd \cos \nu$$

$$-\hat{j}x = -ix \sin A + jx \cos A$$

$$-ky = -iy \cos A \cos \nu - jy \sin A \cos \nu - ky \sin \nu$$

$$\vec{r} = i(-x \sin A - y \cos A \cos \nu - d \cos A \sin \nu) + j(+x \cos A - y \sin A \cos \nu - d \sin A \sin \nu) + k(-y \sin \nu + d \cos \nu)$$

and again from formula (1)

$$\begin{aligned} u &= -x \sin A - y \cos A \cos \nu - d \cos A \sin \nu \\ v &= +x \cos A - y \sin A \cos \nu - d \sin A \sin \nu \\ w &= -y \sin \nu + d \cos \nu \end{aligned} \qquad (3)$$

The coordinates X and Y of a point along the vector $\vec{R}$ for a certain elevation Z are therefore:

$$\begin{aligned} X &= \frac{Z\ (+x \sin A + y \cos A \cos \nu + d \cos A \sin \nu)}{+y \sin \nu - d \cos \nu} \\ Y &= \frac{Z\ (-x \cos A + y \sin A \cos \nu + d \sin A \sin \nu)}{+y \sin \nu - d \cos \nu} \end{aligned} \qquad (4)$$

From Figure 1 we obtain:

$$\begin{aligned} x &= (\bar{x} - \Delta x) \cos \kappa - (\bar{y} - \Delta y) \sin \kappa \\ y &= (\bar{y} - \Delta y) \cos \kappa + (\bar{x} - \Delta x) \sin \kappa \end{aligned} \qquad (5)$$

Substituting these values in (4), we have:

$$X = \frac{Z\left(\left\{\left[(\bar{y}-\Delta y)\cos\kappa + (\bar{x}-\Delta x)\sin\kappa\right]\cos\nu + d\sin\nu\right\}\cos A + \left[(\bar{x}-\Delta x)\cos\kappa - (\bar{y}-\Delta y)\sin\kappa\right]\sin A\right)}{\left[(y-\Delta y)\cos\kappa + (\bar{x}-\Delta x)\sin\kappa\right]\sin\nu - d\cos\nu} \qquad (6)^*$$

---

*These formulas are in agreement with formulas No. 12 derived by O. v. Gruber in Ferienkurs in Photogrammetrie, Page 27.

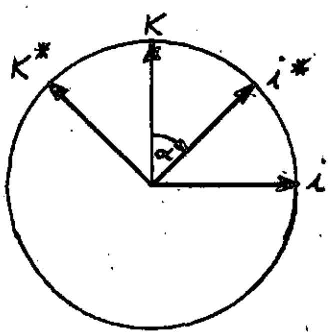
$$\bar{Y} = \frac{Z\left(\left\{\left[(\bar{y}-\Delta y)\cos\kappa + (\bar{x}-\Delta x)\sin\kappa\right]\cos\nu + d\sin\nu\right\}\sin A - \left[(\bar{x}-\Delta x)\cos\kappa - (\bar{y}-\Delta y)\sin\kappa\right]\cos A\right)}{\left[(\bar{y}-\Delta y)\cos\kappa + (\bar{x}-\Delta x)\sin\kappa\right]\sin\nu - d\cos\nu} \qquad (6)\ \text{cont'd}$$

With Z = 1, these formulas reduce to formulas 10a for the standard coordinates ζ, η obtained in BRL Report No. 784 *

Sometimes, especially in aerial photogrammetry, it becomes necessary to express the direction of the camera axis by two tilt angles. Let us denote the tilt in the x-direction by α and the tilt in the y-direction by ω, as in Figure 2. We have:

$$\vec{r} = iu + jv + kw = \hat{i}d + \hat{j}y - \hat{k}x \qquad (7)$$

The individual transformation matrices are:
1.) for a rotation in α-tilt, denoted temporarily by*



$$i = i^* \sin\alpha - k^* \cos\alpha$$

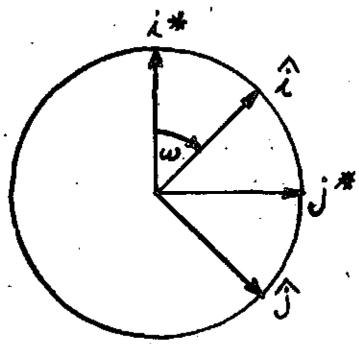
$$j = j^*$$

$$k = i^* \cos\alpha - k^* \sin\alpha$$

$$\begin{array}{c|ccc} & i^* & j^* & k^* \\ \hline i & & & \\ j & & \left(\begin{matrix} \sin\alpha & 0 & -\cos\alpha \\ 0 & 1 & 0 \\ \cos\alpha & 0 & -\sin\alpha \end{matrix}\right) & \\ k & & & \end{array}$$

Rotation clockwise positive

2.) for a rotation in ω-tilt, positive turning clockwise:



$$i^* = \hat{i}\cos\omega - \hat{j}\sin\omega$$

$$j^* = \hat{i}\sin\omega + \hat{j}\cos\omega$$

$$k^* = k$$

$$\begin{array}{c|c} & \begin{matrix} \hat{i} & \hat{j} & \hat{k} \end{matrix} \\ \hline \begin{matrix} i^* \\ j^* \\ k^* \end{matrix} & \left(\begin{matrix} \cos\omega & -\sin\omega & 0 \\ \sin\omega & +\cos\omega & 0 \\ 0 & 0 & 1 \end{matrix}\right) \end{array}$$

*H. Schmid - "Spatial Triangulation by Means of Photogrammetry."

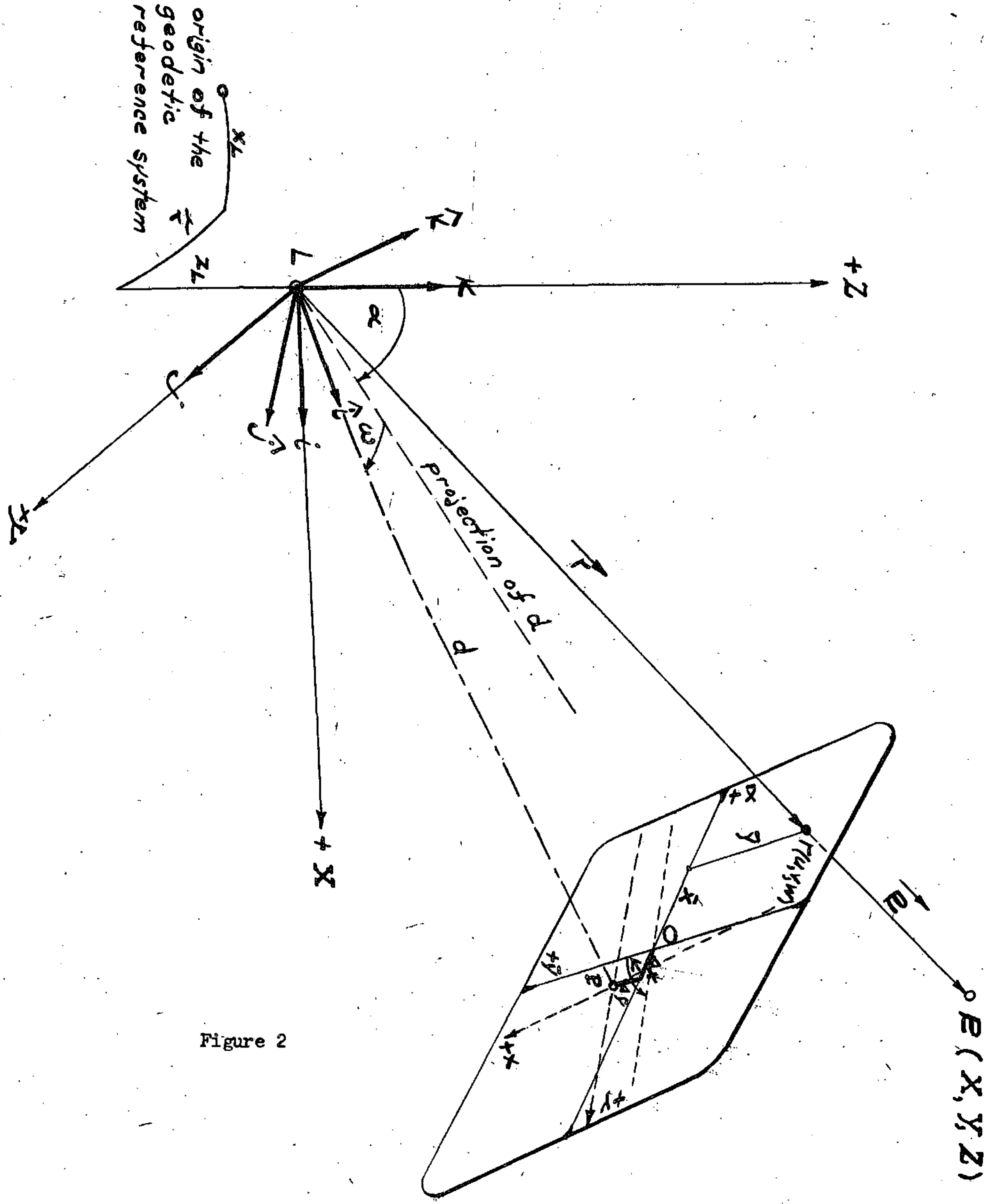

origin of the geodetic reference system
+Z
+X
+Y
projection of d
d
P(X,Y,Z)

Figure 2

and the combined transformation matrix is:

$$
\begin{array}{c|c}
 & \begin{array}{ccc} \hat{i} \qquad\qquad & \hat{j} \qquad\qquad & \hat{k} \end{array} \\
\hline
\begin{array}{c} i \\ j \\ k \end{array} & \begin{pmatrix} \sin\alpha\cos\omega & -\sin\alpha\sin\omega & -\cos\alpha \\ \sin\omega & \cos\omega & 0 \\ \cos\alpha\cos\omega & -\sin\omega\cos\alpha & \sin\alpha \end{pmatrix}
\end{array}
$$

or

$$
\begin{aligned}
\hat{i} &= i\sin\alpha\cos\omega + j\sin\omega + k\cos\alpha\cos\omega, \\
\hat{j} &= -i\sin\alpha\sin\omega + j\cos\omega - k\sin\omega\cos\alpha \\
\hat{k} &= -i\cos\alpha \qquad\qquad\qquad\qquad + k\sin\alpha
\end{aligned} \qquad (8)
$$

with formulas (7) and (8) we obtain:

$$
\begin{aligned}
+\hat{i}d &= +id\sin\alpha\cos\omega + jd\sin\omega + kd\cos\alpha\cos\omega \\
+\hat{j}y &= -iy\sin\alpha\sin\omega + jy\cos\omega - ky\cos\alpha\sin\omega \\
-\hat{k}x &= +ix\cos\alpha \qquad\qquad\qquad\qquad - kx\sin\alpha
\end{aligned}
$$

$$
\begin{aligned}
\vec{r} = {} & i(d\sin\alpha\cos\omega - y\sin\alpha\sin\omega + x\cos\alpha) \\
& + j(d\sin\omega + y\cos\omega) + k(d\cos\alpha\cos\omega - y\cos\alpha\sin\omega - x\sin\alpha)
\end{aligned}
$$

and again with formula (7)

$$
\begin{aligned}
u &= d\sin\alpha\cos\omega - y\sin\alpha\sin\omega + x\cos\alpha \\
v &= d\sin\omega + y\cos\omega \\
w &= d\cos\alpha\cos\omega - y\cos\alpha\sin\omega - x\sin\alpha
\end{aligned} \qquad (9)
$$

The coordinates X and Y for a point along the vector $\vec{R}$ for a certain elevation Z are therefore:

$$X = \frac{Z(d \sin\alpha \cos\omega - y \sin\alpha \sin\omega + x \cos\alpha)}{d \cos\alpha \cos\omega - y \cos\alpha \sin\omega - x \sin\alpha} \tag{10}$$

$$Y = \frac{Z\,(d \sin\omega + y \cos\omega)}{d \cos\alpha \cos\omega - y \cos\alpha \sin\omega - x \sin\alpha}$$

From Figure 2, we obtain

$$\begin{aligned} y &= (\bar{y}-\Delta y)\cos\kappa - (\bar{x}-\Delta x)\sin\kappa \\ x &= (\bar{y}-\Delta y)\sin\kappa + (\bar{x}-\Delta x)\cos\kappa \end{aligned} \tag{11}$$

Substituting formula (11) into formula (10)

$$X = \frac{Z\Big(\big[(\bar{x}-\Delta x)\cos\kappa + (\bar{y}-\Delta y)\sin\kappa\big]\cos\alpha + \big\{d\cos\omega - \big[(\bar{y}-\Delta y)\cos\kappa - (\bar{x}-\Delta x)\sin\kappa\big]\sin\omega\big\}\sin\alpha\Big)}{\big\{d\cos\omega - \big[(\bar{y}-\Delta y)\cos\kappa - (\bar{x}-\Delta x)\sin\kappa\big]\sin\omega\big\}\cos\alpha - \big[(\bar{x}-\Delta x)\cos\kappa + (\bar{y}-\Delta y)\sin\kappa\big]\sin\alpha}$$

(12) *

$$Y = \frac{Z\big\{\big[(\bar{y}-\Delta y)\cos\kappa - (\bar{x}-\Delta x)\sin\kappa\big]\cos\omega + d\sin\omega\big\}}{\big\{d\cos\omega - \big[(\bar{y}-\Delta y)\cos\kappa - (\bar{x}-\Delta x)\sin\kappa\big]\sin\omega\big\}\cos\alpha - \big[(\bar{x}-\Delta x)\cos\kappa + (\bar{y}-\Delta y)\sin\kappa\big]\sin\alpha}$$

For Z = 1 (unit distance), formulas (12) reduce to the expressions given in the BRL Report No. 784** for the standard coordinates ξ and η .

2. Differential Formulas

We will now consider the differential changes on the image vector $\vec{r}$ and object vector $\vec{R}$ respectively. These changes are caused by differential changes of the individual parameters of the solution.

We denote the spatial position of the optical axis with respect to the station system either by the azimuth A and the zenith distance ν, or by the α-tilt and ω-tilt angles. The corresponding differential changes are dA, dν, and dα, dω respectively. The swing angle of the fiducial mark system is denoted by κ and its differential change by d κ . The principal distance of the camera is denoted by d and the coordinates of the origin of the fiducial mark system denoted by O are, in the oriented x,y-system, x_o, and y_o. The corresponding differential changes are dd, dx and dy respectively. The location of any target point on the plate is determined by a fiducial mark system and expressed by $\bar{x}$ and $\bar{y}$; the differential changes being $d\bar{x}$ and $d\bar{y}$. Finally, the spatial location of the center of projection (L) within a certain geodetic reference system is

*Formulas 12 are in agreement with formulas 13 derived by O. v. Gruber. Compare footnote on page 7 .

**cf reference on page 8 .

given by the coordinates x_L, y_L and z_L and the corresponding differential changes are dx_L, dy_L and dz_L .

Coordinates in the various systems according to this nomenclature are summarized in the following table:

				Azimuth, zenith distance-system	α-tilt, ω-tilt-system
ELEMENTS OF ORIENTATION	Exterior	Coordinates of the center of projection	x - coordinate	$x_L \pm dx_L$	$x_L \pm dx_L$
			y - coordinate	$y_L \pm dy_L$	$y_L \pm dy_L$
			z - coordinate	$z_L \pm dz_L$	$z_L \pm dz_L$
		Position angles of the optical axis	Azimuth	$A \pm dA$	α-tilt $\alpha \pm d\alpha$
			Zenith distance	$\nu \pm d\nu$	ω-tilt $\omega \pm d\omega$
			Swing	$\kappa \pm d\kappa$	$\kappa \pm d\kappa$
	Interior	Coordinates of origin of the fiducial mark system	Principal distance	$d \pm dd$	$d \pm dd$
			x - coordinate	$x_o \pm dx$	$x_o \pm dx$
			y - coordinate	$y_o \pm dy$	$y_o \pm dy$
PLANE PLATE COORDINATES		Fiducial mark system	$\bar{x}$ - coordinate	$\bar{x} \pm d\bar{x}$	$\bar{x} \pm d\bar{x}$
			$\bar{y}$ - coordinate	$\bar{y} \pm d\bar{y}$	$\bar{y} \pm d\bar{y}$

A differential translation ds (dx_L, dy_L and dz_L) of the center of projection causes a change in the vector $\vec{r}$, corresponding to the relation $ds = i dx_L + j dy_L + k dz_L$ (13)

The three differential rotations dA, $d\nu$ and $d\kappa$ cause a differential rotation $d\sigma$ on the vector $\vec{r}$. From Figure 1 and 3 we obtain

$$
\begin{aligned}
d\sigma = &+i\,(-d\kappa \sin\nu \cos A + d\nu \sin A) \\
&+j\,(-d\kappa \sin\nu \sin A - d\nu \cos A) \\
&+k\,(+d\kappa \cos\nu + dA)
\end{aligned}
\tag{14}
$$

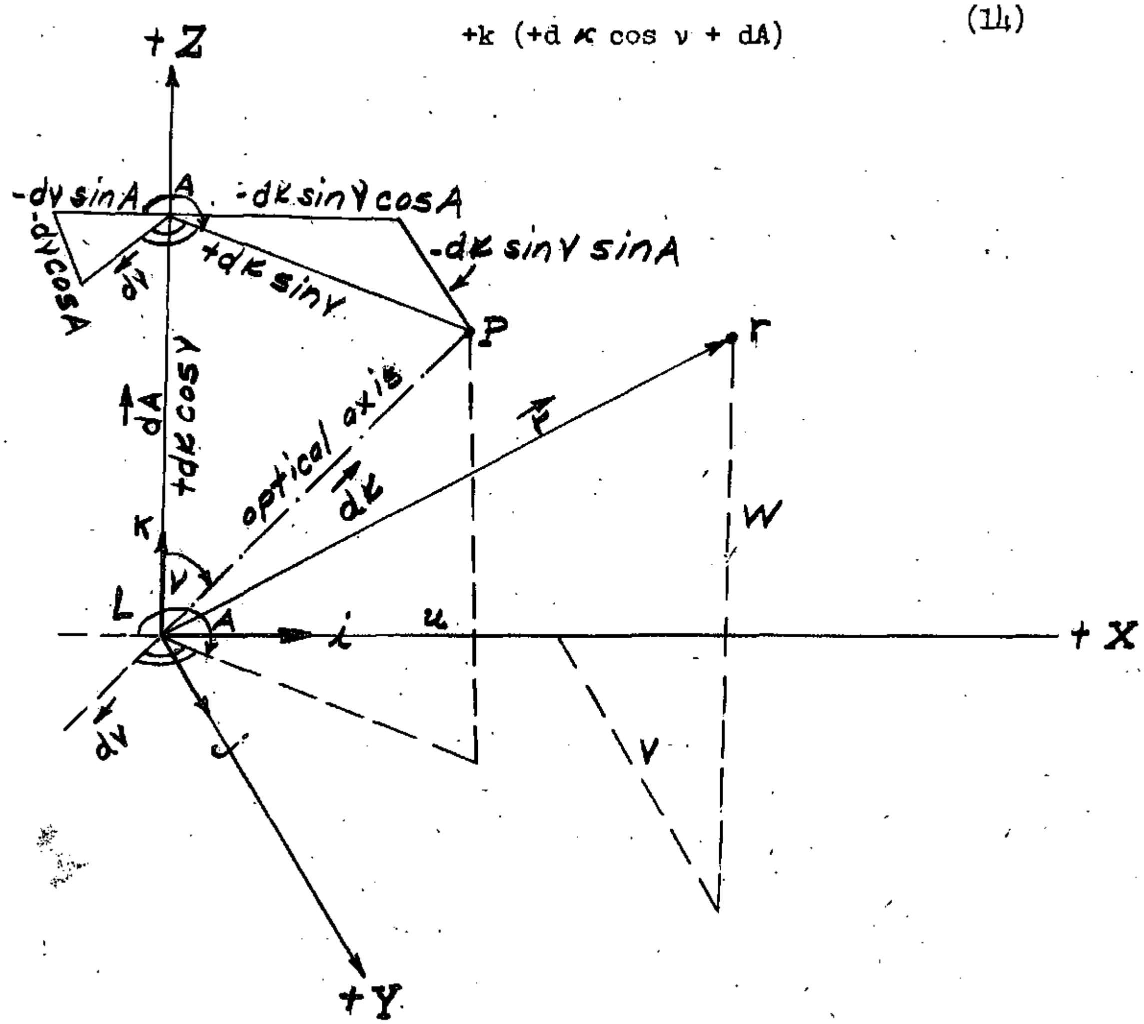


Figure 3

and the corresponding differential rotation in the tilt-system is with Fig. 2 and 4.

$$
\begin{aligned}
d\sigma = &+i(-d\,\kappa \cos\omega \sin\alpha - d\,\omega \cos\alpha) \\
&+j(-d\,\kappa \sin\omega + d\,\alpha) \\
&+k(-d\,\kappa \cos\omega \cos\alpha + d\,\omega \sin\alpha)
\end{aligned} \tag{15}
$$

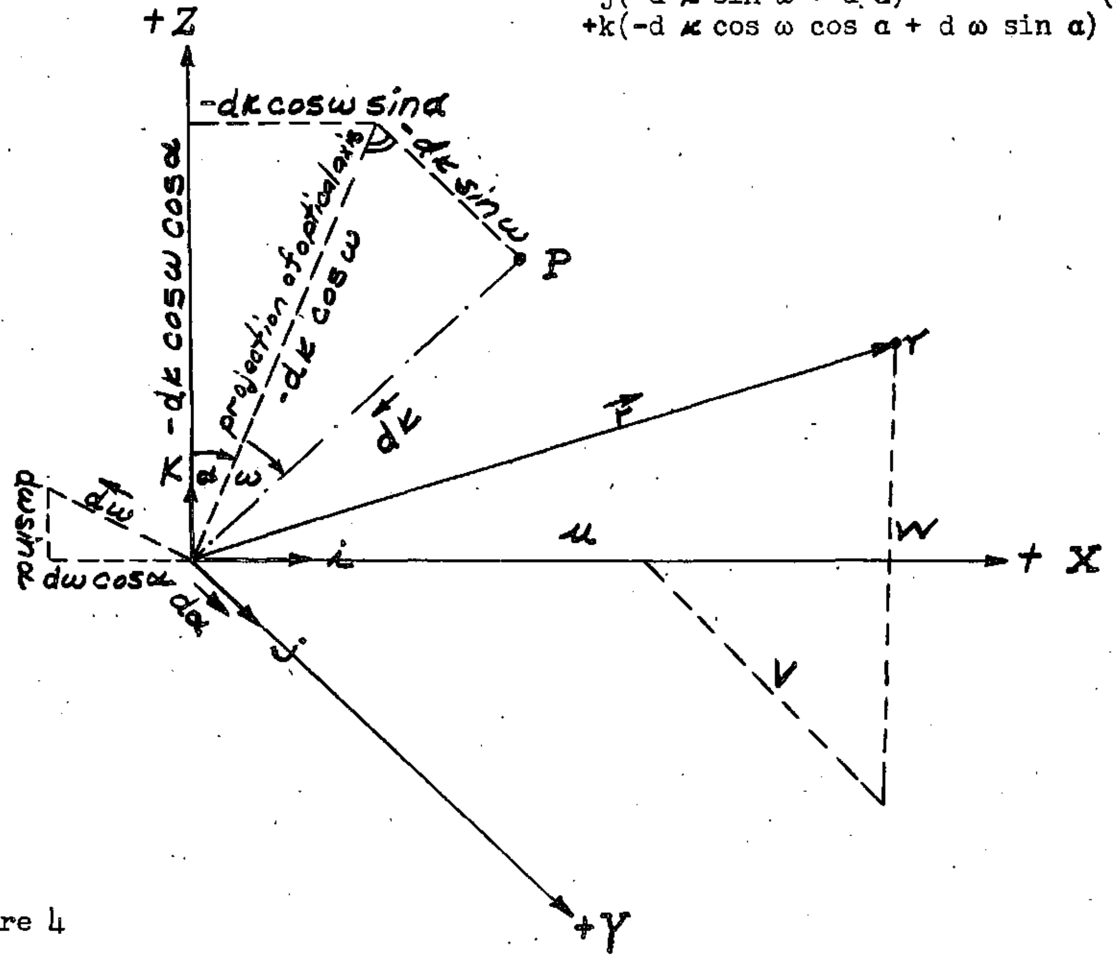


Figure 4

The differential changes on the elements of interior orientation denoted by dd, dx, and dy are three translations in the $\hat{i}, \hat{j}, \hat{k}$ system. With formula 1, we obtain:

$$
\begin{aligned}
(\vec{r} + d\vec{r}) &= i(u + \delta u) + j(v + \delta v) + k(w + \delta w) \\
&= \hat{i}(d + dd) - \hat{j}(x + dx) - \hat{k}(y + dy)
\end{aligned} \tag{16}
$$

and from (3)

$$(\vec{r} + d\vec{r}) = i(u-dd \cos A \sin \nu - dx \sin A - dy \cos A \cos \nu)$$
$$+j(v-dd \sin A \sin \nu + dx \cos A - dy \sin A \cos \nu)$$
$$+k(w+dd \cos \nu - dy \sin \nu) \tag{17}$$

and the corresponding computations with formulas 7 and 9 give for the tilt-system:

$$(\vec{r} + d\vec{r}) = i(u + \delta u) + j(v + \delta v) + k(w + \delta w) = \hat{i}(d+dd)+\hat{j}(y+dy)-\hat{k}(x+dx) \tag{18}$$

$$(\vec{r} + d\vec{r}) = i(u+dd \sin \alpha \cos \omega + dx \cos \alpha - dy \sin \alpha \sin \omega)$$
$$+j(v+dd \sin \omega + dy \cos \omega) \tag{19}$$
$$+k(w+dd \cos \alpha \cos \omega - dx \sin \alpha - dy \cos \alpha \sin \omega)$$

The differential changes $d\bar{x}$ and $d\bar{y}$ are two further translations in the $\hat{i}$, $\hat{j}$, $\hat{k}$ system. With formulas (5) and (11) respectively, we have:

$$d\bar{x} = dy \sin \kappa + dx \cos \kappa$$
$$d\bar{y} = dy \cos \kappa - dx \sin \kappa$$

and

$$d\bar{x} = dx \cos \kappa - dy \sin \kappa$$
$$d\bar{y} = dx \sin \kappa + dy \cos \kappa$$

and therefore, with (17) and (19) respectively

$$d\vec{r} = i\left[-d\bar{x}(+\sin A \cos\kappa + \cos A \cos \nu \sin\kappa)+d\bar{y}(\sin A \sin\kappa - \cos A \cos \nu \cos\kappa)\right]$$
$$+j\left[+d\bar{x}(+\cos A \cos\kappa - \sin A \cos \nu \sin\kappa)-d\bar{y}(+\cos A \sin\kappa+\sin A \cos \nu \cos\kappa)\right] \tag{20}$$
$$+k\left[-d\bar{x} \sin \nu \sin \kappa - d\bar{y}\cdot\sin \nu \cos \kappa\right]$$

and

$$\vec{dr} = i\left[d\bar{x}(\cos\alpha\cos\kappa+\sin\alpha\sin\omega\sin\kappa)+d\bar{y}(\cos\alpha\sin\kappa-\sin\alpha\sin\omega\cos\kappa)\right]$$
$$j\left[-d\bar{x}\cos\omega\sin\kappa+d\bar{y}\cos\omega\cos\kappa\right]$$
$$k\left[d\bar{x}(-\sin\alpha\cos\kappa+\cos\alpha\sin\omega\sin\kappa)-d\bar{y}(\sin\alpha\sin\kappa+\cos\alpha\sin\omega\cos\kappa)\right] \quad (21)$$

The resulting differential change of the vector $\vec{r}$ is obtained by superimposing the individual changes. For A, ν-system, we have with formulas (13), (14), (17), and (20).:

$$\vec{dr} = \left[d\sigma\,\vec{r}\right]+i\left[-dd\cos A\sin\nu-dx\sin A-dy\cos A\cos\nu\right.$$
$$-d\bar{x}(\sin A\cos\kappa+\cos A\cos\nu\sin\kappa)+d\bar{y}(\sin A\sin\kappa-\cos A\cos\nu\cos\kappa)$$
$$\left.+dx_L\right]$$
$$+j\left[-dd\sin A\sin\nu+dx\cos A-dy\sin A\cos\nu+d\bar{x}(\cos A\cos\kappa-\sin A\cos\nu\sin\kappa)\right.$$
$$\left.-d\bar{y}(\cos A\sin\kappa+\sin A\cos\nu\cos\kappa)+dy_L\right] \quad (22)$$
$$+k\left[+dd\cos\nu-dy\sin\nu-d\bar{x}\sin\nu\sin\kappa-d\bar{y}\sin\nu\cos\kappa+dz_L\right]$$

and for the α-ω-tilt system with formulas (13), (15), (19) and (21),

$$\vec{dr} = \left[d\sigma\,\vec{r}\right]+i\left[+dd\sin\alpha\cos\omega+dx\cos\alpha-dy\sin\alpha\sin\omega+d\bar{x}(\cos\alpha\cos\kappa+\sin\alpha\sin\kappa\sin\omega)\right.$$
$$\left.+d\bar{y}(\cos\alpha\sin\kappa-\sin\alpha\cos\kappa\sin\omega)+dx_L\right]$$
$$+j\left[dd\sin\omega+dy\cos\omega-d\bar{x}\cos\omega\sin\kappa+d\bar{y}\cos\omega\cos\kappa+dy_L\right]$$
$$+k\left[dd\cos\alpha\cos\omega-dx\sin\alpha-dy\cos\alpha\sin\omega+d\bar{x}(-\sin\alpha\cos\kappa+\cos\alpha\sin\omega\sin\kappa)\right.$$
$$\left.-d\bar{y}(\sin\alpha\sin\kappa+\cos\alpha\sin\omega\cos\kappa)+dz_L\right] \quad (23)$$

The vector product $\left[d\sigma\,\vec{r}\right]$ in formulas (22) and (23) may be written with formulas (14) and (15) respectively, as:

$$\left[d\sigma\,\vec{r}\right] = \begin{vmatrix} i & j & k \\ -d\kappa\sin\nu\cos A+d\nu\sin A & -d\kappa\sin\nu\sin A-d\nu\cos A & +d\kappa\cos\nu+dA \\ u & v & w \end{vmatrix} \quad (24)$$

and in the tilt system:

$$\left[d\sigma\,\vec{r}\right]=\begin{vmatrix} i & j & k \\ -d\kappa\cos\omega\sin\alpha-d\omega\cos\alpha & -d\kappa\sin\omega+d\alpha & -d\kappa\cos\omega\cos\alpha+d\omega\sin\alpha \\ u & v & w \end{vmatrix} \tag{25}$$

A reduction of these determinants gives for (24)

$$\begin{aligned}\left[d\sigma\,\vec{r}\right]= & \ i\left[-d\kappa(w\sin\nu\sin A+v\cos\nu)-d\nu\cdot w\cos A-dA\cdot v\right]\\ & +j\left[+d\kappa(+u\cos\nu+w\sin\nu\cos A)-d\nu\cdot w\sin A+dA\cdot u\right]\\ & +k\left[-d\kappa(v\sin\nu\cos A-u\sin\nu\sin A)+d\nu(v\sin A+u\cos A)\right]\end{aligned}$$

and for (25)

$$\begin{aligned}\left[d\sigma\,\vec{r}\right]= & +i\left[-d\kappa(w\sin\omega-v\cos\omega\cos\alpha+d\alpha\cdot w-d\omega\cdot v\sin\alpha\right]\\ & +j\left[-d\kappa(u\cos\omega\cos\alpha-w\cos\omega\sin\alpha)+d\omega(u\sin\alpha+w\cos\alpha)\right]\\ & +k\left[-d\kappa(v\cos\omega\sin\alpha-u\sin\omega)-d\alpha\cdot u-d\omega\cdot v\cos\alpha\right]\end{aligned}$$

Consequently, we obtain with formula (16) or (18) from formulas (22) and (23).

$$\begin{aligned}(\vec{r}+d\vec{r})= & +i\big[u-d\kappa(w\sin\nu\sin A+v\cos\nu)-d\nu\cdot w\cos A-dA\cdot v-dd\cos A\sin\nu\\ & \quad -dx\sin A-dy\cos A\cos\nu-d\bar{x}(\sin A\cos\kappa+\cos A\cos\nu\sin\kappa)\\ & \quad +d\bar{y}(\sin A\sin\kappa-\cos A\cos\nu\cos\kappa)+dx_L\big]\\ & +j\big[v+d\kappa(u\cos\nu+w\sin\nu\cos A)-d\nu\cdot w\cdot\sin A+dA\cdot u-dd\sin A\sin\nu\\ & \quad +dx\cos A-dy\sin A\cos\nu+d\bar{x}(\cos A\cos\kappa-\sin A\cos\nu\sin\kappa)\\ & \quad -d\bar{y}(\cos A\sin\kappa+\sin A\cos\nu\cos\kappa)+dy_L\big]\\ & +k\big[w-d\kappa(v\sin\nu\cos A-u\sin\nu\sin A+d\nu(v\sin A+u\cos A)+dd\cos\nu\\ & \quad -dy\sin\nu-d\bar{x}\sin\nu\sin\kappa-d\bar{y}\sin\nu\cos\kappa+dz_L\big]\end{aligned} \tag{26}$$

or

$$
\begin{aligned}
(\vec{r}+\vec{dr}) = {} & +i\Big[u-d\kappa(w \sin\omega - v\cos\omega\cos\alpha)+d\alpha\cdot w - d\omega\cdot v\cdot\sin\alpha + dd\sin\alpha\cos\omega \\
& \quad +dx\cos\alpha - dy\sin\alpha\sin\omega + d\bar{x}(\cos\alpha\cos\kappa+\sin\alpha\sin\kappa\sin\omega) \\
& \quad +d\bar{y}(\cos\alpha\sin\kappa - \sin\alpha\cos\kappa\sin\omega)+dx_L\Big] \\
& +j\Big[v-d\kappa(u\cos\omega\cos\alpha - w\cos\omega\sin\alpha)+d\omega(u\sin\alpha+w\cos\alpha)+dd\sin\omega \\
& \quad +dy\cos\omega - d\bar{x}\cos\omega\sin\kappa + d\bar{y}\cos\omega\cos\kappa + dy_L\Big] \\
& +k\Big[w-d\kappa(v\cos\omega\sin\alpha - u\sin\omega)-d\alpha\cdot u - d\omega\cdot v\cos\alpha + dd\cos\alpha\cos\omega \\
& \quad -dx\sin\alpha - dy\cos\alpha\sin\omega + d\bar{x}(-\sin\alpha\cos\kappa+\cos\alpha\sin\omega\sin\kappa) \\
& \quad -dy(\sin\alpha\sin\kappa+\cos\alpha\sin\omega\cos\kappa)+dz_L\Big]
\end{aligned}
\tag{27}
$$

If we intersect the vector ($\vec{r}+\vec{dr}$) with a plane, we have:

$\lambda(\vec{r}+\vec{dr}) = i\lambda(u+\delta u)+j\lambda(v+\delta v)+k\lambda(w+\delta w)$

For a plane at an elevation w = constant, we obtain,

$$\lambda(w+\delta w) = w$$

or $\lambda = \dfrac{1}{w+\frac{1}{w}\delta w}$ and with sufficient approximation

$$\lambda = (1-\frac{1}{w}\delta w)$$

and, therefore, neglecting second order terms:

$$\lambda(\vec{r}+\vec{dr}) = i(u-\frac{u}{w}\delta w+\delta u) + j(v-\frac{v}{w}\delta w+\delta v) + kw \tag{28}$$

The point in which the vector $\vec{r}+\vec{dr}$ intersects the plane w = constant may be denoted by r'$_3$ and the components of its differential change within this plane are du and dv.

Then

$$\lambda(\vec{r}+\vec{dr}) = i(u+du)+j(v+dv)+kw$$

and consequently with formula (28)

$$du = -\frac{u}{w}\,\delta w + \delta u$$
$$dv = -\frac{v}{w}\,\delta w + \delta v \qquad (29)$$

Further, the object vector $\vec{R}$ may be expressed by

$$\vec{R} = \mu \cdot \vec{r} \qquad (30)$$

where μ is a parameter.

From Figure 1 we obtain:

$$\vec{r} \cdot \hat{i} = d$$

with $\vec{R} \cdot \hat{i} = \mu \cdot \vec{r} \cdot \hat{i}$ we have

$$\mu = \frac{\vec{R}\hat{i}}{d} \qquad (31)$$

and with formulas (1) and (2)

$$\vec{R}\hat{i} = -X \cos A \sin \nu - Y \sin A \sin \nu + Z \cos \nu = D$$
$$\mu = \frac{D}{d} \qquad (32)$$

From formula (30), it follows directly that

$$X = \mu u$$
$$Y = \mu v \qquad (33)$$
$$Z = \mu w$$

Substituting formulas (33) into formula (29), we obtain

$$du = -\frac{X}{Z}\,\delta w + \delta u$$
$$dv = -\frac{Y}{Z}\,\delta w + \delta v \qquad (34)$$

The differential coordinate changes of the point R within a plane Z = constant follow from (30)

$$dX = \mu \, d\, u$$

$$dY = \mu \, d\, v$$

and with (34)

$$\begin{aligned} dX' &= -\mu \frac{X}{Z} \delta w + \mu \delta u \\ dY &= -\mu \frac{Y}{Z} \delta w + \mu \delta v \end{aligned} \qquad (35)$$

The terms for δu, δv and δw are obtained from formulas (26) and (27) with respect to formulas (16) and (18) respectively. Substituting in formula (35), we obtain:

$$\begin{aligned} dX = &-d\kappa \left[-\frac{XY}{Z} \sin \nu \cos A + \left(\frac{X^2}{Z} + Z\right) \sin \nu \sin A + Y \cos \nu\right] \\ &-d\nu \left[+\frac{XY}{Z} \sin A + \left(\frac{X^2}{Z} + Z\right) \cos A\right] \\ &-dA \left[Y\right] \\ &-dd \left[\mu \left(\frac{X}{Z} \cos \nu + \cos A \sin \nu\right)\right] \\ &-dx \left[\mu \sin A\right] \\ &+dy \left[\mu\left(\frac{X}{Z} \sin \nu - \cos A \cos \nu\right)\right] \\ &+d\bar{x} \left[\mu\left(\frac{X}{Z} \sin \nu \sin \kappa - \sin A \cos \kappa - \cos A \cos \nu \sin \kappa\right)\right] \\ &+d\bar{y} \left[\mu\left(\frac{X}{Z} \sin \nu \cos \kappa + \sin A \sin \kappa - \cos A \cos \nu \cos \kappa\right)\right] \\ &-dz_L \left[\frac{X}{Z}\right] \\ &+dx_L \end{aligned} \qquad (36)$$

and

$$
\begin{aligned}
dY = \; & -d\kappa \left[+ \frac{XY}{Z} \sin\nu \sin A - \left(\frac{Y^2}{Z} + Z\right) \sin\nu \cos A - X \cos\nu \right] \\
& -d\nu \left[+ \frac{XY}{Z} \cos A + \left(\frac{Y^2}{Z} + Z\right) \sin A \right] \\
& +dA \left[X \right] \\
& -dd \left[\mu \left(\frac{Y}{Z} \cos\nu + \sin A \sin\nu \right) \right] \\
& +dx \left[\mu \cos A \right] \\
& +dy \left[\mu \left(\frac{Y}{Z} \sin\nu - \sin A \cos\nu \right) \right] \\
& +d\bar{x} \left[\mu \left(\frac{Y}{Z} \sin\nu \sin\kappa + \cos A \cos\kappa - \sin A \cos\nu \sin\kappa \right) \right] \\
& +d\bar{y} \left[\mu \left(\frac{Y}{Z} \sin\nu \cos\kappa - \cos A \sin\kappa - \sin A \cos\nu \cos\kappa \right) \right] \\
& -dz_L \left[\frac{Y}{Z} \right] \\
& +dy_L
\end{aligned}
\qquad (37)
$$

where $\mu = \dfrac{- X \cos A \sin\nu - Y \sin A \sin\nu + Z \cos\nu}{d}$

and the corresponding expressions in the tilt system are:

$$
\begin{aligned}
dX = \; & +d\kappa \left[-\left(\frac{X^2}{Z} + Z\right) \sin\omega + \frac{XY}{Z} \cos\omega \sin\alpha + Y \cos\omega \cos\alpha \right] \\
& +d\alpha \left[\frac{X^2}{Z} + Z \right] \\
& +d\omega \left[\frac{XY}{Z} \cos\alpha - Y \sin\alpha \right] \\
& +dd \left[\mu \left(- \frac{X}{Z} \cos\alpha \cos\omega + \sin\alpha \cos\omega \right) \right] \\
& +dx \left[\mu \left(\frac{X}{Z} \sin\alpha + \cos\alpha \right) \right] \\
& +dy \left[\mu \left(\frac{X}{Z} \cos\alpha \sin\omega - \sin\alpha \sin\omega \right) \right]
\end{aligned}
$$

$$+d\bar{x}\left[\mu\left(\frac{X}{Z}\left\{\sin\alpha\cos\kappa-\cos\alpha\sin\omega\sin\kappa\right\}+\cos\alpha\cos\kappa+\sin\alpha\sin\kappa\sin\omega\right)\right]$$

(38)*

$$+d\bar{y}\left[\mu\left(\frac{X}{Z}\left\{\sin\alpha\sin\kappa+\cos\alpha\sin\omega\cos\kappa\right\}+\cos\alpha\sin\kappa-\sin\alpha\cos\kappa\sin\omega\right)\right]$$

$$-dz_L\left[\frac{X}{Z}\right]$$

$$+dx_L$$

and

$$dY=+d\kappa\left[+\left(\frac{Y^2}{Z}+Z\right)\cos\omega\sin\alpha-\frac{YX}{Z}\sin\omega-X\cos\omega\cos\alpha\right]$$

$$+d\alpha\left[\frac{YX}{Z}\right]$$

$$+d\omega\left[\left(\frac{Y^2}{Z}+Z\right)\cos\alpha+X\sin\alpha\right]$$

$$+dd\left[\mu\left(-\frac{Y}{Z}\cos\alpha\cos\omega+\sin\omega\right)\right]$$

$$+dx\left[\mu\frac{Y}{Z}\sin\alpha\right]$$

$$+dy\left[\mu\left(\frac{Y}{Z}\cos\alpha\sin\omega+\cos\omega\right)\right]$$

$$+d\bar{x}\left[\mu\left(\frac{Y}{Z}\left\{\sin\alpha\cos\kappa-\cos\alpha\sin\omega\sin\kappa\right\}-\cos\omega\sin\kappa\right)\right] \qquad (39)^*$$

$$+d\bar{y}\left[\mu\left(\frac{Y}{Z}\left\{\sin\alpha\sin\kappa+\cos\alpha\sin\omega\cos\kappa\right\}+\cos\omega\cos\kappa\right)\right]$$

$$-dz_L\left[\frac{Y}{Z}\right]$$

$$+dy_L$$

here:

$$\mu=\frac{+X\sin\alpha\cos\omega+Y\sin\omega+Z\cos\alpha\cos\omega}{d}$$

---

* The coefficients in formulas (38) and (39) for the differentials of the elements of exterior orientation agree with the results obtained by A. Brandenberger in Fehlertheorie der äusseren Orientierung von Steilaufnahmen, Zürich, 1946.

With $Z = 1$ (unit distance) the formulas (36), (37), (38), and (39) reduce to the formulas for the differential changes $d\xi$ and $d\eta$ of the corresponding standard coordinates ξ and η.*

---

* BRL Report No. 784 cf reference on p. 8.

III. THE TRIANGULATION OF THE MOST PROBABLE POINT AND THE PROPAGATION OF RESIDUAL ERRORS IN THE TRIANGULATION PROCEDURE

In intersection photogrammetry, we have in general as many "rays" to the target point as there are established measuring stations. Due to the inevitable errors, these rays will not intersect in space unless corrections are applied to the individual rays. The criterion selected for the most probable corrections must lead to the determination of the most probable point of intersection. First we shall assume that the systematic errors of the orientation elements are small compared with the expected accuracy of the plate measurements and can, therefore, be neglected. This means that we assume that the elements of interior and exterior orientation are known sufficiently accurately either from dial readings and instrument calibration procedures, or from preceding computations of the orientation elements from recorded control points. Consequently, we may say that the most probable point is determined by the intersection of such rays for which the corrections applied to the measured plate coordinates of the individual measuring stations satisfy the condition that the sum of the least squares is a minimum. After the most probable corrections have been applied we have a rigorous triangulation and the spatial coordinates may now be computed from any two stations.

Formulas for such a triangulation follow:

In Figure 5 we have two stations - I and II - for which $\vec{P}_1 = \vec{b} + \vec{P}_2$. This, multiplied vectorially by $\vec{p}_2$ gives $\vec{P}_1 \times \vec{p}_2 = \vec{b} \times \vec{p}_2 + \vec{P}_2 \times \vec{p}_2$ and because $\vec{P}_2 \times \vec{p}_2 = 0$, we obtain with $\vec{P}_1 = \mu \cdot \vec{p}_1$ (40)

where μ is a parameter.

$$\mu = \frac{\vec{b} \times \vec{p}_2}{\vec{p}_1 \times \vec{p}_2} .$$

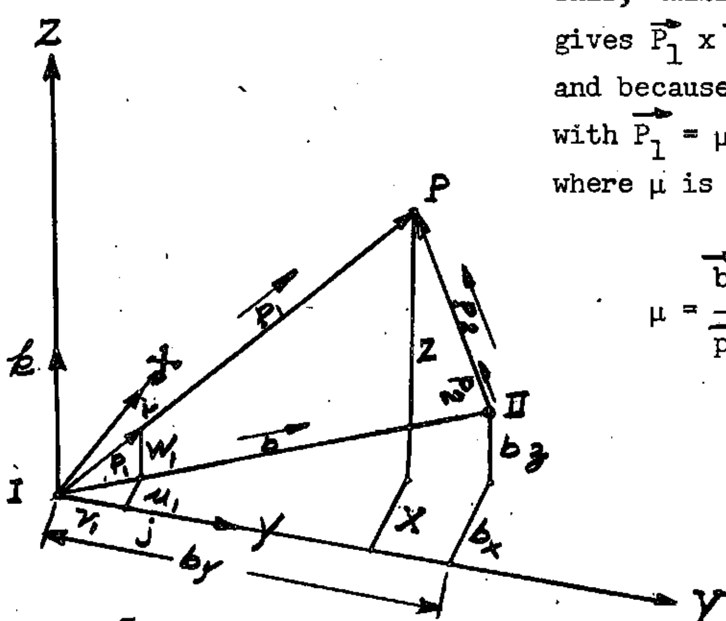


Fig.5.

With

$$\vec{p_1} = iu_1 + jv_1 + kw_1$$

$$\vec{p_2} = iu_2 + jv_2 + kw_2$$

$$\vec{b} = ib_x + jb_y + kb_2$$

the vector products are:

$$\vec{b} \times \vec{p_2} = \begin{vmatrix} i & j & k \\ b_x & b_y & b_z \\ u_2 & v_2 & w_2 \end{vmatrix}$$

$$\vec{p_1} \times \vec{p_2} = \begin{vmatrix} i & j & k \\ u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \end{vmatrix}$$

The directions of both vector products are the same and consequently

$$\mu = \frac{\begin{vmatrix} b_x & b_y \\ u_2 & v_2 \end{vmatrix}}{\begin{vmatrix} u_1 & v_1 \\ u_2 & v_2 \end{vmatrix}} = \frac{\begin{vmatrix} b_y & b_z \\ v_2 & w_2 \end{vmatrix}}{\begin{vmatrix} v_1 & w_1 \\ v_2 & w_2 \end{vmatrix}} = \frac{\begin{vmatrix} b_x & b_z \\ u_2 & w_2 \end{vmatrix}}{\begin{vmatrix} u_1 & w_1 \\ u_2 & w_2 \end{vmatrix}} \qquad (41)$$

Directly from formula (40) :

$$X = \mu_1 u_1 = \mu_2 u_2$$

$$Y = \mu_1 v_1 = \mu_2 v_2 + b$$

$$Z = \mu_1 w_1 = \mu_2 w_2 + b_Z \qquad (42)$$

The u, v and w values are computed from formulas (3) or (9). It is noted that the vector triplet of the Station II must first be transformed in the vector triplet of the Station I in order to eliminate the earth's curvature.

Although the result obtained with (42) is correct only for a unique solution, these formulas enable us to study the propagation of the error of the individual stations in a triangulation procedure. We shall limit our consideration to two stations. Further, we shall neglect the influence of the curvature of the earth and consider the errors on the geodetic basic data as negligibly small. We introduce a cartesian coordinate system with its origin at Station I oriented in such a way that the horizontal base line component coincides with the + y axis, as shown in Figure 6.

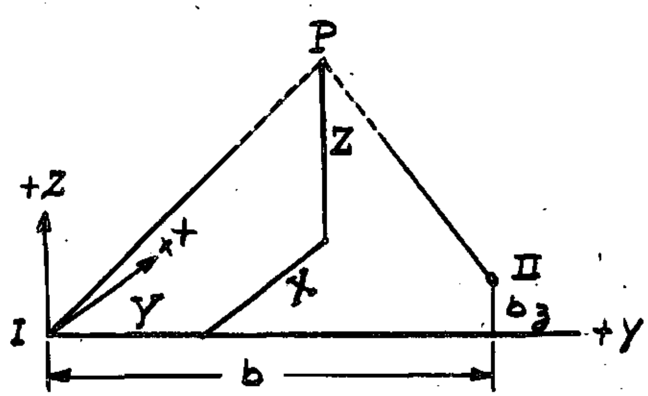


Fig. 6

From formulas (42), we obtain by differentiation, the differential changes in the space coordinates, δX, δY and δZ.

$$\delta X = \mu_1 \, \delta u_1 + u_1 \, \delta \mu_1$$

$$\delta Y = \mu_1 \, \delta v_1 + v_1 \, \delta \mu_1$$

$$\delta Z = \frac{\mu_1 \, \delta w_1 + w_1 \, \delta \mu_1 + \mu_2 \, \delta w_2 + w_2 \, \delta \mu_2}{2} \qquad (43)$$

With formulas (41), (42) and (43)

$$\delta X = \frac{Y\mu_1\delta u_1-(Y-b)\mu_2\delta u_2-X(\mu_1\delta v_1-\mu_2\delta v_2)}{b}$$

$$\delta Y = \frac{\frac{Y(Y-b)}{X}(\mu_1\delta u_1-\mu_2\delta u_2)-(Y-b)\mu_1\delta v_1+Y\mu_2\delta v_2}{b} \tag{44}$$

$$\delta Z = \frac{Z(2Y-b)-b_zY}{2bX}(\mu_1\delta u_1-\mu_2\delta u_2)-\frac{2Z-b_z}{2b}(\mu_1\delta v_1-\mu_2\delta v_2)+\frac{1}{2}(\mu_1\delta w_1+\mu_2\delta w_2)$$

where δu_i, δv_i and δw_i are obtained from formulas (26) or (24).
From formulas (42) and (26), we have:

$$\begin{aligned}
\mu_i\delta u_i = & -Y_i\cdot dA_i \\
& -Z_i \cos A_i d\nu_i \\
& -(Z_i \sin \nu_i \sin A_i+Y_i \cos \nu_i)\cdot d\kappa_i \\
& -\mu_i \sin A_i\cdot dx_i \\
& -\mu_i \cos A_i \cos \nu_i\cdot dy_i \\
& -\mu_i (\sin A_i \cos \kappa_i+\cos A_i \cos \nu_i \sin \kappa_i)d\bar{x}_i \\
& +\mu_i (\sin A_i \sin \kappa_i-\cos A_i \cos \nu_i \cos \kappa_i)d\bar{y}_i \\
& -\mu_i \cos A_i \sin \nu_i\cdot dd_i
\end{aligned}$$

$$\begin{aligned}
\mu_i\delta v_i = & +XdA_i \\
& -Z \sin A_i\cdot d\nu_i \\
& +(X \cos \nu_i+Z_i \sin \nu_i \cos A_i)\cdot d\, \kappa_i \\
& + \mu_i \cos A_i\cdot dx_i \\
& -\mu_i \sin A_i \cos \nu_i\, dy_i
\end{aligned}$$

$$+\mu_i(\cos A_i \cos \kappa_i - \sin A_i \cos \nu_i \sin \kappa_i)\cdot d\bar{x}_i$$

$$-\mu_i(\cos A_i \sin \kappa_i + \sin A_i \cos \nu_i \cos \kappa_i)\cdot d\bar{y}_i$$

$$-\mu_i \sin A_i \sin \nu_i \cdot dd_i$$

$$\mu_i \delta w_i = +(Y_i \sin A_i + X \cos A_i)\cdot d\nu_i$$

$$-(Y_i \cos A_i \sin \nu_i - X_i \sin A_i \sin \nu_i)\cdot d\kappa_i$$

$$-\mu_i \sin \nu_i \cdot dy_i$$

$$-\mu_i \sin \nu_i \sin \kappa_i \cdot d\bar{x}_i$$

$$-\mu_i \sin \nu_i \cos \kappa_i \cdot d\bar{y}_i$$

$$+\mu_i \cos \nu_i \cdot dd_i$$

where, in accordance with formulas (31) and (32),

$$\mu_i = \frac{D_i}{d_i} \text{ and } D_i = -X_i \cos A_i \sin \nu_i - Y_i \sin A_i \sin \nu_i + Z_i \cos \nu_i$$

and from formula (27)

$$\mu_i \delta u_i = +Z_i d\alpha_i$$

$$-Y_i \sin \alpha_i \cdot d\omega_i$$

$$-(Z_i \sin \omega_i - Y_i \cos \omega_i \cos \alpha_i)\cdot d\kappa_i$$

$$+\mu_i \cos \alpha_i \cdot dx_i$$

$$-\mu_i \sin \alpha_i \sin \omega_i \cdot dy_i$$

$$+\mu_i (\cos \alpha_i \cos \kappa_i + \sin \alpha_i \sin \kappa_i \sin \omega_i)\cdot d\bar{x}_i$$

$$+\mu_i (\cos \alpha_i \sin \kappa_i - \sin \alpha_i \cos \kappa_i \sin \omega_i)\cdot d\bar{y}_i$$

$$+\mu_i \sin\alpha_i \cos\omega_i \cdot dd_i$$

$$\mu_i \delta v_i = +(X \sin\alpha_i + Z_i \cos\alpha_i) d\omega_i$$

$$-(X \cos\omega_i \cos\alpha_i - Z_i \cos\omega_i \sin\alpha_i) \cdot d\kappa_i$$

$$+\mu_i \cos\omega_i \cdot dy_i$$

$$-\mu_i \cos\omega_i \sin\kappa_i \cdot d\bar{x}_i$$

$$+\mu_i \cos\omega_i \cos\kappa_i \cdot d\bar{y}_i$$

$$+\mu_i \sin\omega_i \cdot dd_i$$

$$\mu_i \delta w_i = -X d\alpha_i$$

$$-Y_i \cos\alpha_i \, d\omega_i$$

$$-(Y_i \cos\omega_i \sin\alpha_i - X_i \sin\omega_i) \cdot d\kappa_i$$

$$-\mu_i \sin\alpha_i \cdot dx_i$$

$$-\mu_i \cos\alpha_i \sin\omega_i \, dy_i$$

$$-\mu_i (\sin\alpha_i \cos\kappa_i - \cos\alpha_i \sin\omega_i \sin\kappa_i) \cdot d\bar{x}_i$$

$$-\mu_i (\sin\alpha_i \sin\kappa_i + \cos\alpha_i \sin\omega_i \cos\kappa_i) \cdot d\bar{y}_i$$

$$+\mu_i \cos\alpha_i \cos\omega_i \cdot dd_i$$

where, with formulas (31) and (32),

$$\mu_i = \frac{D_i}{d_i} \text{ and } D_i = X \sin\alpha_i \cos\omega_i + Y_i \sin\omega_i + Z_i \cos\alpha_i \cos\omega_i.$$

Hellmut Schmid.

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